

*Perspective***Risk-Based Sampling: I Don't Want to Weight in Vain****Mark R. Powell**

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Recently, there has been considerable interest in developing risk-based sampling for food safety and animal and plant health for efficient allocation of inspection and surveillance resources. The problem of risk-based sampling allocation presents a challenge similar to financial portfolio analysis. Markowitz (1952) laid the foundation for modern portfolio theory based on mean-variance optimization. However, a persistent challenge in implementing portfolio optimization is the problem of estimation error, leading to false “optimal” portfolios and unstable asset weights. In some cases, portfolio diversification based on simple heuristics (e.g., equal allocation) has better out-of-sample performance than complex portfolio optimization methods due to estimation uncertainty. Even for portfolios with a modest number of assets, the estimation window required for true optimization may imply an implausibly long stationary period. The implications for risk-based sampling are illustrated by a simple simulation model of lot inspection for a small, heterogeneous group of producers.

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**KEY WORDS:** Risk-based sampling; sanitary and phytosanitary risk

**1. INTRODUCTION**

Recently, there has been considerable interest in developing scientific schemes for risk-based sampling of food, animals, and plants for effective enforcement of regulatory standards and efficient allocation of surveillance resources. It seems intuitive that products and producers that present higher sanitary and phytosanitary (SPS) risks warrant higher frequency and intensity of safety inspection and surveillance activities performed under a budget constraint. Indeed, this resource allocation problem presents a challenge similar to the familiar problem where an investor seeks to optimize the allocation of limited funds among alternative assets. However, both the SPS and finance domains are characterized by non-stationary processes, and this presents a fundamental challenge for optimization strategies.<sup>(1-3)</sup> After introducing some background on risk-based sampling in the SPS context, this perspective will briefly review some relevant findings from the financial portfolio

optimization literature. This is followed by a simple simulation model that illustrates some of the implications for risk-based sampling in the SPS arena.

There appears to be widespread agreement, at least in principle, that risk-based sampling represents an optimal strategy for resource allocation. Beginning in the early 1990s, policy reviews have recommended that federal food safety agencies adopt risk-based inspection.<sup>(4,5)</sup> In practice, the development and implementation of risk-based food safety inspection has been a challenge, particularly in light of legal constraints on minimum inspection frequencies and the limitations of available data to make risk-based distinctions among food products, food producers, or hazards associated with foods.<sup>(6-12)</sup> Appropriate tools and metrics are available for risk ranking of biological hazards.<sup>(13)</sup> However, a commonly cited limitation in applying these tools is the magnitude of uncertainty given the available data. Uncertainty about dose-response relationships looms large, for example, and epidemiologically-based foodborne illness attribution data are not available for the vast majority of pathogen-food combinations.<sup>(14)</sup> For foodborne toxicants, Finkel<sup>(15)</sup>

U.S. Department of Agriculture, Office of Risk Assessment and Cost Benefit Analysis, 1400 Independence Ave., SW (MS 3811), Washington, DC 20250, USA; mpowell@oce.usda.gov.

illustrates that generally accepted risk rankings (e.g., the risk from aflatoxin in peanut butter is greater than the risk from Alar in apple juice) can become indiscernible after taking uncertainty fully into account.

In the SPS setting, interest in risk-based inspection is not limited to food safety. Recently, the idea also has caught on in animal and plant health. However, Stark<sup>(16)</sup> cautions: “The rapid rate of acceptance of this core concept of risk-based [animal health] surveillance has outpaced the development of its theoretical and practical bases.” Williams<sup>(17)</sup> considers the advantages, pitfalls, and ambiguities in targeted sampling for animal disease surveillance. This approach differs from stratified and other common sampling approaches in that samples can be drawn exclusively from targeted subpopulations, and inferences rely on auxiliary epidemiologic information used to estimate risk ratios and demographics, which are themselves subject to uncertainty. Similar to some of the implementation challenges witnessed in food safety, an initial effort by the U.S. Department of Agriculture Animal Plant and Health Inspection Service (USDA/APHIS) to introduce risk-based sampling for imports of plants for planting was put on temporary hold shortly after its introduction in 2012.<sup>(18)</sup> Conceptually, risk-based sampling presents a challenge similar to financial portfolio optimization, although the SPS domain is data-poor relative to finance. Prattley<sup>(19)</sup> and Cannon<sup>(20)</sup> provide examples of the application of financial portfolio theory to animal health surveillance. More generally, Cox<sup>(21)</sup> has recommended application of portfolio optimization methods to manage any portfolio of risks.

## 2. FINANCIAL PORTFOLIO OPTIMIZATION: THEORY AND PERFORMANCE

Over 60 years ago, Markowitz<sup>(22)</sup> laid the foundation for modern portfolio theory by deriving the optimal rule for allocating wealth among assets in a single-period setting when an investor bases decisions only on the mean and variance of a portfolio’s return. This involves calculating the weights ( $w$ ) allocated to  $N$  different assets in a portfolio to minimize the variance of returns on the portfolio ( $\sigma_p^2$ ) for a given expected return ( $\mu_p$ ), or equivalently to maximize  $\mu_p$  for a given  $\sigma_p^2$ . Such an investor chooses a portfolio along the mean-variance efficient frontier based on his or her target expected return ( $\mu_p^*$ ) or degree of “risk aversion.” (The financial literature typically equates risk with variance.<sup>(23)</sup>) The mean-

variance efficient frontier can be traced by solving:<sup>1</sup>

$$\text{Min}_w w' \Sigma w = \sigma_p^2 \quad (1)$$

$$\text{s.t. } w' \mu = \mu_p^*, w' \mathbf{1} = 1$$

$$\text{where: } w = \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix}, \quad \Sigma = \begin{bmatrix} \sigma_1^2 & \cdots & \sigma_{1N} \\ \vdots & \ddots & \vdots \\ \sigma_{N1} & \cdots & \sigma_N^2 \end{bmatrix},$$

$\mu = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_N \end{bmatrix}$ ,  $\mathbf{1}$  denotes a vector of ones, and  $\mu_p^*$  is a specified value.

In the absence of a constraint prohibiting short sales (i.e., absent  $w_i \geq 0 \forall i$ ),<sup>2</sup> the solution for  $w$  is given by:

$$\mathbf{A}^{-1} \begin{bmatrix} 0 \\ \mu_p^* \\ 1 \end{bmatrix} = \begin{bmatrix} w \\ \lambda_1 \\ \lambda_2 \end{bmatrix} \quad (2)$$

where:  $\mathbf{A} = \begin{bmatrix} 2 \Sigma & \mu & \mathbf{1} \\ \mu' & 0 & 0 \\ \mathbf{1}' & 0 & 0 \end{bmatrix}$  and  $\lambda_1$  and  $\lambda_2$  are Lagrange multipliers.

To trace the efficient frontier under Equation (2),  $\mu_p^*$  is varied by increments. In the single-period setting, the investor holds  $w$  fixed over the period  $[t, t+1]$ , after which the weights may be adjusted.

The key insight of Markowitz’s theory is that for a given expected return, variance is reduced by holding a portfolio of imperfectly correlated assets. It also provided a theoretically appealing explanation for the observed behavior of investors who

<sup>1</sup>An alternative formulation of the classical MVO problem includes the tradeoff between mean and variance directly through the objective function:  $\text{Max}_w w' \mu - \lambda w' \Sigma w$ , where  $\lambda \geq 0$  is a risk aversion parameter.

<sup>2</sup>Short sales are sales of an asset not owned by the seller who expects to buy it at a lower price at a future date and result in exposure to potentially unlimited losses. Short sales enter the model as negative weights. Brennan and Lo<sup>(24)</sup> show that in the absence of estimation error, the probability that the mean-variance efficient frontier contains negative weights tends to 1 as the number of assets in a portfolio increases without bound. This is inconsistent with the capital asset pricing model (CAPM), under which negative asset weights are impossible. The CAPM holds that the portfolio in which each asset’s weight is proportional to its total market capitalization lies on the mean-variance-efficient frontier.

seek diversification rather than simply maximizing expected returns. At about the same time, Roy<sup>(25)</sup> noted that an investor's objective may be to minimize the probability that the portfolio return is less than a specified value. For the case where the specified minimum return is the rate of return on a risk-free asset ( $R_f$ ),<sup>3</sup> both methods lead to the same strategy of maximizing the Sharpe ratio  $((\mu_p - R_f)/\sigma_p)$ , resulting in the tangency portfolio:<sup>(26)</sup>

$$\mathbf{w} = \frac{\boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}_e}{\mathbf{1}' \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}_e} \quad (3)$$

where:  $\boldsymbol{\mu}_e = \boldsymbol{\mu} - R_f \mathbf{1}$ .

Despite its theoretical appeal and simple, elegant solution, mean-variance optimization (MVO) has not gained wide acceptance in the investment community.<sup>(27)</sup><sup>4</sup> Institutional pension portfolios, for example, are anchored to a traditional 60/40 equity/bond benchmark structure.<sup>(29)</sup> Markowitz himself followed a simple investment rule: "I split my contributions 50/50 between bonds and equities."<sup>(30)</sup> Various cognitive, institutional, technological, and other barriers to adoption of classical MVO and its various extensions have been posited.<sup>(26,27,31-33)</sup> For instance, Benartzi<sup>(33)</sup> found that in contrast to the "rational, mean-variance optimizing investor," individual retirement investors tend to employ a naïve diversification strategy, the "1/N heuristic," in which contributions are simply divided evenly among the  $N$  options offered. Finance practitioners also are considered suspicious of portfolios that are not naively diversified.<sup>(34)</sup> As we will see shortly, however, the practitioners' suspicions have some empirical basis.

A fundamental problem underlying MVO is the assumption of perfect information about  $\boldsymbol{\mu}$  and  $\boldsymbol{\Sigma}$  for a future time period ( $t+1$ ) for the universe of assets under consideration. In practice, the true, unknown  $\boldsymbol{\mu}_{t+1}$  and  $\boldsymbol{\Sigma}_{t+1}$  are estimated by  $\hat{\boldsymbol{\mu}}_t$  and  $\hat{\boldsymbol{\Sigma}}_t$  based on information available at time  $t$ . Consequently, when MVO is applied, the optimization problem that is actually solved is:

$$\text{Min}_{\mathbf{w}} \mathbf{w}' \sum_{t+1} \mathbf{w} + \mathbf{w}' \left( \hat{\boldsymbol{\Sigma}}_t - \sum_{t+1} \right) \mathbf{w} \quad (4)$$

<sup>3</sup>Risk-free rate of return refers to zero variance, e.g., the rate of return on a short-term government-issued security with zero risk of default.

<sup>4</sup>More generally, Black and Litterman<sup>(28)</sup> remark that "few global investment managers regularly allow quantitative models to play a major role in their asset allocation decisions."

$$\text{s.t. } \mathbf{w}' \boldsymbol{\mu}_{t+1} + \mathbf{w}' (\hat{\boldsymbol{\mu}}_t - \boldsymbol{\mu}_{t+1}) = \mu^*, \mathbf{w}' \mathbf{1} = 1$$

where  $(\hat{\boldsymbol{\Sigma}}_t - \boldsymbol{\Sigma}_{t+1})$  and  $(\hat{\boldsymbol{\mu}}_t - \boldsymbol{\mu}_{t+1})$  are *ex ante* unobservable estimation errors.

MVO using sample-based parameter estimates drawn from a cross-sectional time series of historical returns assumes that the future is drawn from the same multivariate distribution as the past. For stationary processes like rolling dice, estimation error due to finite samples can, in principle, be rendered negligible. In nonstationary processes like financial markets, however, there are regime shifts (e.g., in macroeconomic conditions), transients (e.g., natural disasters and geopolitical disruptions), and other complex dynamics. Consequently, arbitrarily increasing the sample size does not arbitrarily improve the precision of forecasts.<sup>(35)</sup> The spread and evolution of infectious agents and invasive species in the SPS setting are similarly nonstationary processes. Indeed, there is a rapidly growing literature emphasizing and exploiting the parallels between financial and ecological and infectious disease risks.<sup>(36-40)</sup>

MVO using sample-based inputs is notorious for producing extreme, unstable asset weights<sup>5</sup> and for exhibiting poor out-of-sample performance.<sup>(42-47)</sup> The optimization seeks to exploit the slightest apparent differences among assets. For example, assume a portfolio consists of five identical assets with monthly returns  $(r) \sim \text{Normal}(1\%, 4\%)$ , correlations of 50%, and  $R_f = 0\%$ . The true optimal  $\mathbf{w}' = [0.2, \dots, 0.2]$ .<sup>6</sup> A simple simulation shows, however, that if we obtain  $\hat{\boldsymbol{\mu}}$  and  $\hat{\boldsymbol{\Sigma}}$  from a sample of 120 months, the weights calculated by the optimization are highly

<sup>5</sup>Extreme weights result in undiversified portfolios concentrated in a small number of assets and may represent large leveraged positive allocations and/or negative short allocations. Weights that are unstable over time result in high transaction costs. In principle, it is straightforward to incorporate future transaction costs associated with adjusting to a new efficient frontier into the objective function:  $\text{Max}_{\mathbf{w}} \mathbf{w}' \boldsymbol{\mu} - \lambda \mathbf{w}' \sum \mathbf{w} - C_t$ , where  $C_t$  is the transaction costs (including taxes) resulting from sales of existing and purchases of new positions. However, portfolio optimization models with transaction costs remain a challenge analytically and empirically.<sup>(41)</sup> For example, predicting transaction costs would require predictable price changes that in theory are rapidly eliminated in informationally efficient markets. In practice, transaction costs may be limited indirectly by introducing a constraint on asset turnover, the sum of the absolute differences in weights between adjacent periods. In studies evaluating the performance of portfolio allocation methods, turnover generally serves as a proxy measure for transaction costs.

<sup>6</sup>In general, the 1/N portfolio is mean-variance efficient if  $\boldsymbol{\mu} \propto \boldsymbol{\Sigma} \mathbf{1}$ . This includes the case of identical assets.

volatile about the mean of 0.2 (95% confidence interval  $-0.47$ – $0.87$ ).

While the growing interest in the application of portfolio optimization methods in the SPS domain is a relatively recent development, the limitations of modern portfolio theory have long been recognized in the financial field. Frankfurter,<sup>(48)</sup> Hodges,<sup>(49)</sup> and Dickenson<sup>(50)</sup> considered the sensitivity of MVO to estimation error. Frankfurter<sup>(48)</sup> suggested that under realistic conditions, portfolios selected according to mean-variance criteria are no more likely to be efficient than portfolios selected at random. Dickenson<sup>(50)</sup> remarked that “the practical results ... are sufficiently poor for the investment analyst to be forgiven for relying on his intuition.” Barry<sup>(51)</sup> cautioned that due to nonstationary asset return distributions, the amount of information available from a given historical time series would be limited.

Merton<sup>(52)</sup> showed that under idealized conditions, volatility can be measured precisely using high frequency data, but very long time series would be needed to estimate expected returns with precision. Merton also cautioned that even if long time series are available, it may not be reasonable to assume stationary parameters over that long period. (Even under stationary conditions, relatively new assets by definition provide a short time series. Google, for example, was founded in 1998.<sup>7)</sup> Michaud<sup>(27)</sup> demonstrated that statistically equivalent portfolios can have very different asset weights and argued that given estimation uncertainty, the optimal portfolio is not well-defined. For example, addressing parameter uncertainty by resampling, the distribution of weights for a given asset in a portfolio can be bimodal, with the asset frequently excluded from the efficient portfolio.<sup>(53,54)</sup> Best<sup>(55)</sup> presented theoretical results showing that portfolio weights are sensitive to small perturbations in the means, and that the sensitivity of the weights increases with the number of assets in a portfolio and the correlation among assets. Chopra<sup>(56)</sup> showed that the relative impact of estimation errors in means, variances, and covariances depends on the position along the mean-variance efficient frontier.

Michaud<sup>(27)</sup> also noted that in practice, an important reason for the instability of MVO solutions is the inversion of an ill-conditioned covariance matrix (e.g., due to high dimensionality and/or multicollinearity). A simple example illustrates. Assume

a portfolio consists of three assets (A, B, and C) with the following monthly returns ( $r$ ) and correlations ( $\rho$ ):  $r_A \sim \text{Normal}(1\%, 2\%)$ ,  $r_B \sim \text{Normal}(1.05\%, 2.5\%)$ ,  $r_C \sim \text{Normal}(1\%, 3\%)$ ,  $\rho_{AB} = 0.9$ ,  $\rho_{AC} = 0.5$ , and  $\rho_{BC} = 0.1$ . Let  $R_f = 0\%$ . Under this scenario, the solution for  $\mathbf{w}' = [13\%, 52\%, 35\%]$ , which appears reasonable. Now, holding all else constant, let  $\rho_{AC} = 0.523$ . Under this scenario, the solution for  $\mathbf{w}' = [-2899\%, 2089\%, 910\%]$ . This nonsensical combination of extreme long and short positions is the result of an ill-conditioned covariance matrix. Note that  $\Sigma$  can be decomposed into matrices of eigenvectors ( $\mathbf{v}_i$ ) and eigenvalues ( $\lambda_i$ ), and  $\mathbf{w}$  is a linear combination of  $N$  principal portfolios  $\mathbf{v}_i$  with weights that scale as  $\lambda^{-1}$ :

$$\mathbf{w} \propto \sum_{i=1}^N \frac{1}{\lambda_i} \mathbf{v}_i \mathbf{v}_i' \mu_e \quad (5)$$

where:  $\sum_{i=1}^N \frac{1}{\lambda_i} \mathbf{v}_i \mathbf{v}_i' = \Sigma^{-1}$ .

Consequently, MVO aligns the weights with the principal portfolios linked with small eigenvalues.<sup>(57)</sup> With the slight change of  $\rho_{AC}$  (0.5 to 0.523), the smallest  $\lambda$  decreases by more than an order of magnitude to 0.0016. As the eigenvectors associated with the smallest eigenvalues are most sensitive to noise, Michaud<sup>(27)</sup> dubbed this the “error maximization” property of mean-variance optimizers.

Over the past 60 years, researchers have busily developed new methods devoted to improving the performance of portfolio optimization by reducing the impact of estimation error and relaxing underlying assumptions (e.g., no transaction costs or taxes, unlimited liquidity, joint elliptically distributed returns, constant volatility and linear dependencies, a market with no memory).<sup>(26,58)</sup> For example, index models imposed structure on the correlation matrix and reduced the dimensionality of the optimization problem.<sup>(59)</sup> (However, this involves a tradeoff between estimation error and specification error.<sup>(60)</sup>) Barry<sup>(51)</sup> recommended using diffuse Bayesian priors to address estimation error. Jobson<sup>(42,47)</sup> and Jorion<sup>(43,61)</sup> proposed empirical Bayes shrinkage estimators. Black<sup>(28)</sup> proposed combining two priors, shrinking the views of the investor toward an equilibrium asset pricing model, depending on the degree of confidence in the investor’s views. Robust portfolio allocation rules seek to minimize opportunity costs under a “reasonable set” of market scenarios reflecting parameter and model uncertainty.<sup>(62,63)</sup>

Among practitioners, a popular method for controlling the effects of estimation error and instability is to impose *ad hoc* constraints, such as turnover

<sup>7</sup>Similarly, in the SPS domain, individual producers or exporting nations may have a short track record.

limits or minimum and maximum position limits on individual assets or asset classes.<sup>(41,58,64)</sup> A common constraint is to impose nonnegative weights (prohibiting short sales). According to Brennan,<sup>(24)</sup> “investment professionals...have railed against mindless optimization for years, arguing that portfolio weights obtained in this manner are ill-behaved and must be constrained or otherwise post-processed.” Ang<sup>(64)</sup> remarks, “[c]onstraints help because they bring back unconstrained portfolio weights to economically reasonable positions.” The common practice of relying on *ad hoc* constraints raises the question of whether the optimized solution is driven by subjective views of what the optimal assets weights “should be” and has led some observers to suggest that as used currently, MVO “has largely a marketing, rather than investment function”<sup>(65)</sup> and serves as “window dressing.”<sup>(66)8</sup>

Over 30 years ago, Bloomfield<sup>(67)</sup> and Jobson<sup>(42)</sup> observed that the naïve  $1/N$  portfolio strategy can outperform MVO and other sophisticated allocation strategies. More recently, DeMiguel<sup>(45)</sup> employed a 10-year rolling estimation window to compare the out-of-sample performance of the naïve  $1/N$  strategy to 14 portfolio optimization models across seven empirical data sets of monthly returns. The models include classical MVO and its extensions like Bayesian estimators and parameter restrictions, as well as asset pricing models, constrained portfolios, and combinations of portfolios intended to reduce the effects of estimation error. The allocation strategies were compared in terms of the Sharpe ratio, certainty-equivalent return, and turnover. The results indicate that despite decades of increased methodological sophistication, none of the portfolio optimization methods consistently had better out-of-sample performance than the naïve  $1/N$  portfolio. Furthermore, DeMiguel<sup>(45)</sup> estimated that for a portfolio containing 25–50 assets, the estimation window needed for optimization methods to outperform the  $1/N$  strategy is approximately 3,000–6,000 months (250–500 years).

It is an empirical question whether the forecasting error inherent in portfolio optimization methods outweighs the effects of ignoring information under simple allocation rules such as  $1/N$ . The balance is situational, depending on the assets under consid-

<sup>8</sup>Similarly, minimum and maximum prescribed inspection frequencies may constrain risk-based sampling in the SPS setting, and complex risk-ranking algorithms may convey an impression of rigor despite weak underlying data.

**Table I.**  $3 \times 3$  Factorial for 27 Simulated Producers

Factor	High	Med	Low
Mean lot prevalence ( $\mu$ )	0.01	0.005	0.001
CV lot prev. ( $\sigma/\mu$ )	2	1	0.5
Volume ( $L$ , lots/year)	100,000	10,000	1,000

eration, the investment period, the degree of risk aversion, and other factors. Assuming markets are reasonably efficient, however, it seems likely that if an asset allocation strategy were superior in practice, it would have overtaken competing strategies in 60 years. Once the assumption of perfect information about the distribution of future states of the world is relaxed, there is little reason to expect that optimization is an optimal strategy.<sup>(68)</sup>

### 3. SIMULATION OF LOT INSPECTION

The implications of the limitations of portfolio optimization for risk-based sampling are illustrated by a simple simulation model of lot inspection for a small, heterogeneous group of producers. Assume a heterogeneous group of 27 producers characterized by three factors with three levels (Table I). Let  $i = 1, \dots, 27$  producers;  $j = 1, \dots, 20$  years. For each producer, the prevalence of contaminated lots ( $p$ ) varies independently year to year following a beta distribution:<sup>9</sup>

$$p_i \sim \text{Beta}(\mu_i, \sigma_i), \quad (6)$$

where:  $\sigma_i = cv_i * \mu_i$  and  $cv$  is the coefficient of variation ( $\sigma/\mu$ ).

While prevalence has to be estimated, producer volume ( $L_i$ , lots/year) is considered known and fixed. Total volume over all 27 producers is 999,000 lots/year. Assume a budget that allows lot inspection ( $I$ ) of approximately 1% of the total annual volume ( $\sum_{ij} I_i \approx 9,990/\text{year}$ ). Let the probability of detecting a contaminated lot ( $p_{\text{detxn}} = 78.5\%$ ). (This would hold if the within-lot prevalence is 5% and the number of samples per lot is 30. In that case, the budget would permit collection and analysis of approximately 299,700 samples per year.<sup>10</sup>)

<sup>9</sup>Here, we define the beta distribution in terms of the mean ( $\mu$ ) and variance ( $\sigma^2$ ). It is more common to parameterize the beta distribution in terms of  $\alpha$  and  $\beta$  with  $\alpha = \mu^2(1-\mu)/\sigma^2$  and  $\beta = \alpha(1-\mu)/\mu$ .

<sup>10</sup>In comparison, the U.S. Department of Agriculture Food Safety and Inspection Service<sup>(69)</sup> analyzes approximately 30,000

The simulation of contaminated lots ( $c$ ), contaminated lots included in the inspection sample ( $ci$ ), and contaminated lots detected upon inspection ( $x$ ) proceeds as follows:

$$\begin{aligned}
 \text{no. contaminated lots } (c_{ij}) &\sim \text{Binomial}(L_i, p_{ij}) \\
 \text{no. contaminated lot inspected } (ci_{ij}) \\
 &\sim \text{Hypergeometric}(L_{ij}, ci_{ij}, L_i) \\
 \text{no. contaminated lots detected } (x_{ij}) \\
 &\sim \text{Binomial}(ci_{ij}, p_{\text{detxn}})
 \end{aligned} \tag{7}$$

Under a constrained, “risk-based” optimization of lot inspection, we start in the first year with an equal number of inspections to establish a baseline estimate of prevalence for each producer. Thereafter, the annual frequency of lot inspection for each producer ( $I_{ij}$ ) is at least one and no more than their production volume, but otherwise sampling is proportional to the product of volume ( $L_i$ ) and the estimated prevalence of contaminated lots ( $\hat{p}_{ij}$ ). The prevalence estimate is updated each year based on the accumulated data:

$$\text{Optimized Allocation : } I_{ij} \propto L_i * \hat{p}_{ij} \tag{8}$$

$$s.t. I_{i1} = 9, 990/27 = 370; 1 \leq I_{ij} \leq L_i \text{ for } j = 2, \dots, 20$$

$$\text{where: } \hat{p}_{ij} = \frac{\sum_{j=1}^{i-1} x_i}{\sum_{j=1}^{i-1} L_i}$$

Under a simpler allocation rule, we ignore information about prevalence and sample lots for inspection proportional to volume.

$$\text{Simplified Allocation : } I_i \propto L_i \tag{9}$$

Note that under simple random sampling, the probability of inspecting a lot from a producer is proportional to its production volume. If production volume were unknown, the simplified allocation rule could be approached by random sampling of lots for inspection.

Two simulation scenarios are considered. Under the first scenario, the system is assumed stationary for 20 years. Under the second scenario, we introduce transients (e.g., outbreaks or extreme contamination events) into an otherwise stationary process. This scenario assumes a producer’s annual probability of a transient is 5% so that each producer is expected to incur one transient over 20 years. The transients are assumed to increase lot prevalence

samples per year for Salmonella across the entire raw meat and poultry sector.

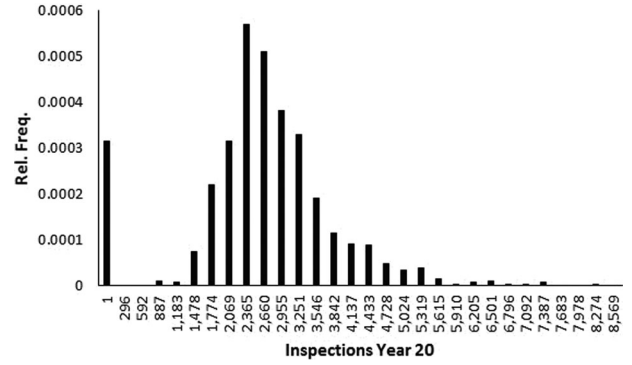


Fig. 1. Optimized allocation distribution for Producer 3 (high volume, high prevalence, low CV) year 20.

without changing the probability of detecting a contaminated lot.<sup>11</sup> Simulations were performed with Latin hypercube sampling using Palisades @Risk, Ver. 5.7.1, an add-on to Microsoft Excel.

### 3.1. Simulation Results

#### 3.1.1. Scenario 1

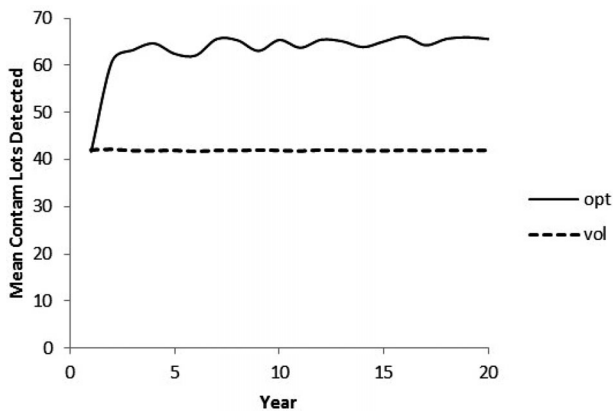
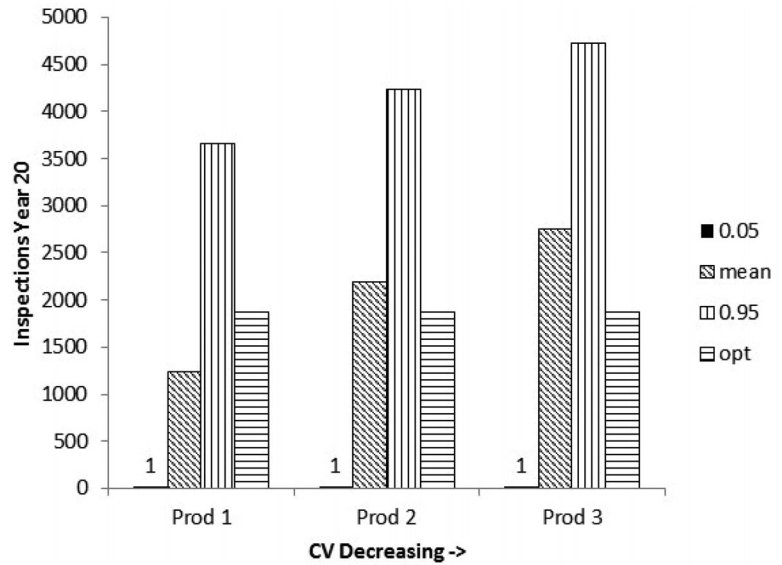
Assuming stationary parameters, the optimized “risk-based” sampling frequencies assigned to producers remain highly unstable after 20 years. For example, Fig. 1 presents the results for Producer 3 (high volume, high prevalence, low cv). After 20 years, the optimization allocates a bimodal distribution of inspections, essentially indicating that the producer should either be ignored or sampled with a high frequency.

Fig. 2 summarizes the optimized allocation distributions for the three high-volume, high-prevalence producers (Producers 1–3). The risk-based sampling weights remain highly unstable after 20 years and far from optimal. Whether the simulated mean under- or overestimates the true optimal allocation depends on the cv.

Even if the simulated baseline sampling period is extended to 10 years, the risk-based sampling weights remain highly unstable at year 20 under stationarity. For example, with a 10-year baseline, the simulated 95% interval of inspections at year 20 for Producer 1 was 29–184% of the true optimum. Just as unstable

<sup>11</sup>This assumption might hold, for example, if contamination becomes more widespread among sites (e.g., orchards) that supply raw materials (e.g., fruit) but not more prevalent within lots sourced from contaminated sites after pre- and postharvest control measures, which could include quarantine of highly contaminated sites.

**Fig. 2.** Simulated mean and 90% confidence intervals of optimized allocation distributions for high-prevalence, high-volume producers year 20.



**Fig. 3.** Mean number of contaminated lots detected over time by optimized and volume-based allocations under stationarity.

asset weights increase financial transaction costs and reduce net portfolio returns, we would expect unstable sampling weights to increase costs and diminish cost effectiveness of lot inspection. Unstable weights also could result in churning reputation damages as different producers are targeted over time.

Fig. 3 compares the performance of the optimization rule versus allocating proportional to volume alone. It suggests that although the optimization is unstable, it is expected to outperform simple volume-based allocation over time. This is analogous to the finding that financial portfolio returns can be less sensitive to estimation error than the underlying asset weights.<sup>(55)</sup> Recall, however, that this simulation sce-

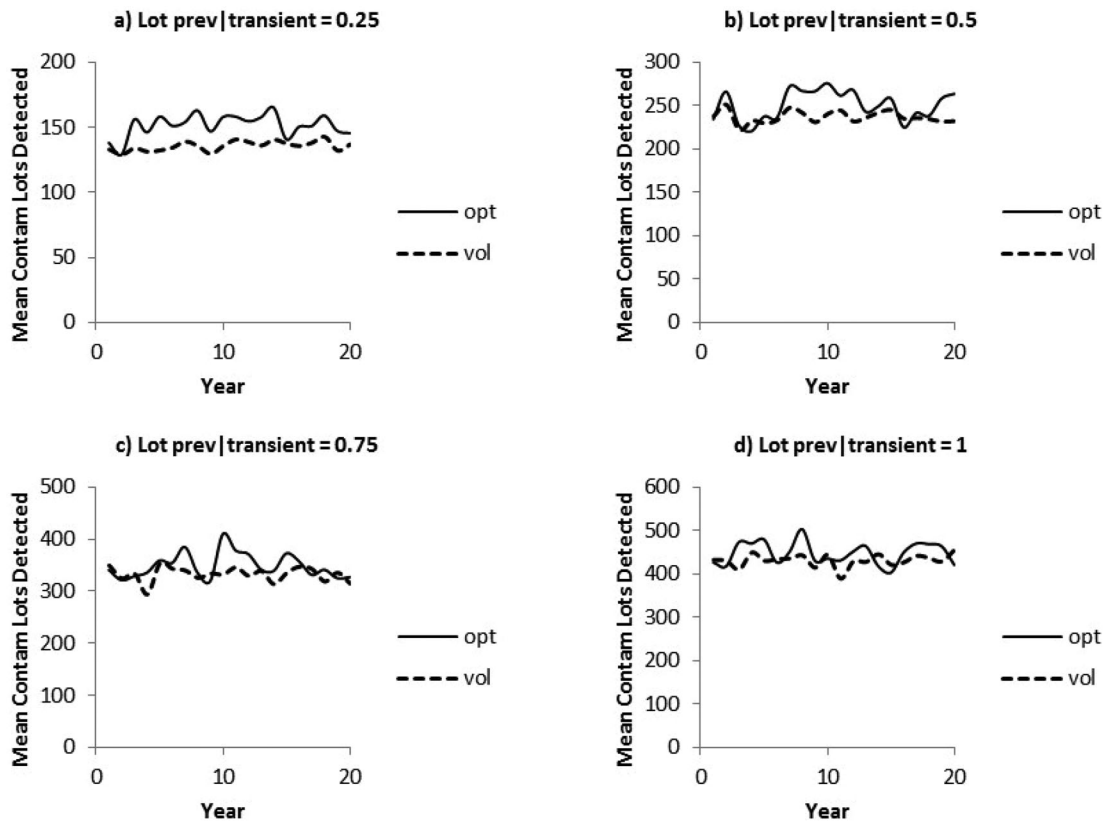
nario implausibly assumes the process is stationary over 20 years.

### 3.1.2. Scenario 2

Fig. 4 shows that with the introduction of infrequent transients into an otherwise stationary process, the expected performance of the allocation rules rapidly becomes indistinguishable as the lot prevalence given a transient (transient intensity) increases.

It is important to note that measuring performance based on the number of lots rejected per year, like classical MVO, assumes a single-period setting. However, rather than passively recording spiking numbers and adjusting the sampling weights at the end of a period, SPS inspection ideally also serves to detect ongoing extreme events, determine their cause, and take timely corrective action. This is not only relevant to food safety but also to invasive species. Once introduced into a new environment, invasive species may become established and cause continuing damages beyond the initial introduction event.

To consider which sampling allocation rule is more likely to detect the occurrence of a transient, Fig. 5 presents the cumulative distribution of the ratio of contaminated lots detected for the two schemes given occurrence of a transient ( $x_{opt}/x_{vol}|transient$ ). We see that the optimization scheme is like a batter who swings for the fences and strikes out a lot.



**Fig. 4.** Mean number of contaminated lots detected over time by optimized and volume-based allocations with producer’s annual probability of transient = 5%.



**Fig. 5.** Cumulative distribution of the ratio of contaminated lots detected under optimization and volume-based allocation given occurrence of a transient.

The break-even ratio of 1 exceeds the 70th percentile of the distribution, and the optimization scheme frequently fails to detect any contaminated lots during transient events due to the virtual exclusion of some producers from inspection. While the risk-based sampling weights are volatile, reacting to each new chunk

of data, the simple volume-based sampling weights are patiently stable over time. To make another sports analogy, the simple scheme is like experienced soccer players who space themselves out over the field, rather than pee wee leaguers who merrily chase the ball as a swarm.



Of course, these simulations are based on hypothetical distributions and scenarios and are only intended to be illustrative. But the results suggest that complex optimization efforts do not consistently outperform apparently naïve allocation strategies. This mirrors the empirical findings in the financial portfolio literature, which is and likely will remain a far more data-rich environment than the SPS setting.

#### 4. CONCLUSION

This article illustrates that seeking to optimize risk-based sampling can be a suboptimal sampling strategy. More generally, Gigerenzer<sup>(70)</sup> finds that heuristic decision making that ignores some information can lead to more accurate judgments than weighting and incorporating all available information, for instance, for conditions characterized by low predictability and small samples. Further, it is worth noting that all samples are small if the dimension of the problem is sufficiently high (e.g., rank ordering the risk of all producers of all foods across all hazards). But if pursuing optimization can be suboptimal under uncertainty, it does not follow that following simple rules is costless or that any simple heuristic will perform better than optimization methods. For example, an investor could simply allocate her wealth among assets based on offers in her email spam folder, or an SPS agency could allocate inspection resources based on the latest widely publicized outbreak or contamination event. In finance, the simple  $1/N$  investment heuristic often performs well in practice because it has built-in buy low/sell high characteristics, a contrarian strategy that benefits from reversion to the mean. Equal weighting also guarantees that the best performing asset remains in the portfolio while eliminating exposure to risky short positions. Recently, Pflug<sup>(32)</sup> demonstrates that optimized portfolios converge to the uniform portfolio as uncertainty increases.

This finding is intuitive. If uncertainty about asset returns is sufficiently large, we would not reject the hypothesis that all the assets arise from the same population. In this instance, applying MVO results in equal weighting, and diversification benefits are achieved if the assets are imperfectly correlated. Similarly, in the SPS setting, if there is sufficient uncertainty (taking into account the number of comparisons), we would not reject the hypothesis that all producers have the same prevalence of contaminated lots. In such a case, sampling resources should be allocated according to risk-related attributes over

which producers are distinguishable (e.g., volume) without regard to prevalence. Alternatively, if the risks associated with exposure pathways are poorly understood (including future volumes), then it will be difficult for risk-weighted sampling to consistently perform better out sample than simple random sampling. This conclusion seems so self-evident as to be trivial. Nonetheless, enthusiasm for risk-based sampling using ever more complicated optimization techniques does not appear to be waning.

In a widely publicized speech on financial regulation in the wake of the financial crisis that began in 2007, Haldane<sup>(68)</sup> advocated the use of simple regulatory decision-making rules under uncertainty but cautioned that this places a heavy reliance on the judgment of the decisionmaker to pick appropriate heuristics. Exercising judgment under uncertainty is taken for granted in private finance. However, relying on the judgment of public decisionmakers assumes a level of deference that is not always granted. And this may explain, at least in part, the persistent appeal of risk-based sampling in the SPS domain.

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